Make sure to explain the grammars and PDAs that you construct. You will be graded both on your construction, as well as your explanation.
1. Let the grammar \( G = (V, \Sigma, P, \langle \text{STMT} \rangle) \) be as follows.

\[ \Sigma = \{ \text{if, condition, then, else, } a = 1 \} \]
\[ V = \{ \langle \text{STMT} \rangle, \langle \text{ASSIGN} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle \} \]

Production set \( P \) is

\[ \langle \text{STMT} \rangle \rightarrow \langle \text{ASSIGN} \rangle | \langle \text{IF-THEN} \rangle | \langle \text{IF-THEN-ELSE} \rangle \]
\[ \langle \text{IF-THEN} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle \]
\[ \langle \text{IF-THEN-ELSE} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle \]
\[ \langle \text{ASSIGN} \rangle \rightarrow a = 1 \]

(a) Show that the grammar \( G \) is ambiguous.
(b) Give a new grammar that generates the same language as \( G \).

2. Give a grammar for each of the following languages. You must explain how the grammar works. If the grader does not understand your grammar or explanation you will receive 0 points.

(a) \( L = \) the set of strings over \( \{a, b\}^* \) that are not palindromes.
(b) Strings not of the form \( \text{ww} \) over alphabet \( \{a, b\}^* \). Possible hint: \( i + j + i + j = i + i + j + j \)

3. Draw a PDA for each of the following languages. You must explain how the PDA works. If the grader does not understand your PDA or explanation you will receive 0 points.

(a) \( \{ a^i b^j c^k \mid j = 2i \text{ or } k = 2j \} \).
(b) \( L = \) the complement of \( \{(a^i b^j)^* \mid 1 < j < i\} \). For example, the string \( aababaabbaabbaabb \) is not in \( L \) because it is of the form \( (a^3 b^2)^* \). But the strings \( a^3 b^3 a^3 b^3, aabaaabb, \) and \( bab \) are in \( L \) because they are not of the form specified.

4. Define \( A@B = \{ xy \mid x \in A, y \in B, \text{ and } |x| = |y| \} \). Prove that if \( A \) and \( B \) are regular, then \( A@B \) is context-free.

5. For each \( i \geq 0 \), let \( \mathcal{L}_i \) be the class of languages accepted by PDAs \( M \) such that \( M \), on any input, never has more than \( i \) symbols pushed onto its stack. Thus, \( \mathcal{L}_i \) are those languages accepted by some PDA that has maximum stack depth \( i \). Answer the following, and prove your answers correct.

(a) What is \( \mathcal{L}_0 \)?
(b) What is the relationship between \( \mathcal{L}_{13} \) and \( \mathcal{L}_{148} \)? (Be careful.)
(c) What is \( \bigcup_{i=1}^{\infty} \mathcal{L}_i \)?