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1. Prove that recursive languages and recursively enumerable languages are both closed under Kleene *. That is, show that if \( L_1 \) is any recursive language, and \( L_2 \) is any recursively enumerable language, then \( L_1^* \) is recursive, and \( L_2^* \) is recursively enumerable.

2. Let \( L = \{ i \mid L(M_i) \text{ contains some string of even length}\} \). Prove that \( L \) is recursively enumerable but not recursive. In proving that it is not recursive, do not use Rice's theorem. You must give a reduction from some known nonrecursive language. What can you now conclude about the language \( \{ i \mid L(M_i) \text{ contains only strings of odd length}\} \)? Why?

3. Decidable or undecidable? Prove that your answer is correct.
   (a) Given a one-tape TM \( M \) and a word \( w \), decide whether or not \( M \) ever moves its head three times to the right consecutively when run on \( w \).
   (b) Given a one-tape TM \( M \) and a word \( w \), decide whether or not \( M \) ever moves its head to the left when run on \( w \).

4. For each of the following decision problems, define the language that represents the decision problem, and show either that
   (i) the language is not recursively enumerable, or
   (ii) the language is recursively enumerable but is not recursive
       i.e., the decision problem is undecidable. (You may not invoke Rice's theorem.), or
   (iii) the language is recursive (i.e., the decision problem is decidable).
   (a) To decide whether or not a program (i.e., a TM) accepts any prime number.
   (b) To decide if a program (i.e., a TM) accepts the set of prime numbers (written in decimal).
   (c) To decide if a program (i.e., a TM) gets 100% on MP3 of CS 225.