Chapter 1

1. What is an ill-conditioned problem?
2. What happens to a stable algorithm to a well-conditioned problem?
3. Are floating point number equally distributed? What does this imply about $a + b$ and $b + a$?
4. What is underflow versus machine epsilon?
5. What is backward error and how do you calculate it for a small problem?
6. What is the relationship between forward and backward error?
7. What is cancellation? Find an example where rearranging a mathematical computation results in lower cancellation.
8. If you have a sequence of numbers, is it better to sum them from smallest to largest, or from largest to smallest? Does it depend on the sign? Why?

Chapter 2

1. What is the norm of a matrix in different norms?
2. What is the condition number and what does it measure?
3. What is the relationship between the condition number and the singular or eigenvalues of a matrix?
4. What is pivoting and is it dependent on the conditioning of the matrix?
5. When does an LU factorization or a Cholesky factorization exist?
6. Why is a symmetric and positive definite matrix useful to have?
7. Will Gaussian Elimination always result in a solution with a small residual?
8. If $A$ has a condition number of $10^{10}$, how many digits of accuracy can you expect Gaussian Elimination to accurate to?
9. What is the initial pivot element of the matrix

$$
\begin{bmatrix}
4 & -8 & 1 \\
6 & 5 & 7 \\
0 & -10 & -3
\end{bmatrix}
$$

using no pivoting, partial pivoting, complete pivoting?
10. What is the cost of Gaussian Elimination?
11. What is the cost of a forward and backward solve?
12. Give an example of a matrix that has a condition number of 1 that is not the identity.
13. Does a small residual always imply an accurate solution? When and why?
14. What are some advantages of a Cholesky factorization?
Chapter 3

1. Does a linear least squares problem always have a solution?
2. What is the relationship between the residual and the \( \text{span}(A) \) at the least-squares solution?
3. What is an orthogonal matrix?
4. Orthogonalize two vectors using Gram-Schmidt.
5. Orthogonalize two vectors using Householder.
6. Orthogonalize two vectors using Givens Rotations.
7. Try a least-squares fit for a simple problem.
8. If \( C \) is orthogonal and \( \|x\|_2 = 5.0 \), then what is \( \|Cx\|_2 \)?
9. What is a QR factorization and how does it relate to a least-squares problem?
10. What is the advantage/disadvantage of solving the normal equations for a least-squares problem?

Chapter 4

1. Is the conditioning of an eigenvalue problem the same as that of solving a linear system?
2. Are all matrices diagonalizable? What does that mean?
3. Run one step of power method.
4. Run one step of inverse power method.
5. What does the power/inverse power method compute?
6. What is the QR iteration?
7. What does a shift in a method accomplish?