Combinational Design

Your friend\(^1\) is rather picky about what pizzas they are willing to eat. From your observations, they will only eat pizzas that conform to the following rules:

1. The pizza should have exactly one vegetable (either mushrooms or peppers).
2. The pizza should not have bacon unless it also has mushrooms.
3. The pizza shouldn’t have both sausage and peppers.

Write a boolean expression, \(L\), that evaluates to true for any combination of ingredients that your friend is willing to eat on pizza. \((Use\ B = \text{bacon},\ M = \text{mushrooms},\ P = \text{peppers},\ S = \text{sausage.})\) A blank truth table is provided for your convenience.

\(L =\)

\[\begin{array}{cccccccccccccccc}
\hline
B & M & P & S & L \\
\hline
1 & 0 & 0 & 0 & \\
0 & 1 & 0 & 0 & \\
0 & 0 & 1 & 0 & \\
0 & 0 & 0 & 1 & \\
\end{array}\]

\(^1\text{FWIW, my own rules for pizza are much simpler – it must only have cheese, and lots of it}\)
Compiler Magic

GCC’s optimizer converts the absolute value function, where an `int` is 32 bits wide:

```c
int abs(int num) {
    if (num >= 0) {
        return num;
    } else {
        return -num;
    }
}
```

into the following:

```c
int abs(int num) {
    int temp = num >> 31;
    return (num ^ temp) - temp;
}
```

You’ll learn why this is an optimization when we talk about pipelining and branch prediction. For now, explain why this works. Recall that `^` is the bitwise XOR operator, and how 2’s complement negation and right shifts on signed quantities work.
Bit-wise Fun

Given an 8-bit signed integer $x$, write expressions using only bit-wise logical and shifting operators to do the following:

- Check if $x$ is odd (i.e., the expression should evaluate to 0 when $x$ is even and 1 when $x$ is odd).

- Invert only the least significant bit of $x$.

- Set bit 2 of $x$ (setting a bit means making it 1, and recall that the least significant bit is numbered bit 0).

- Clear bit 1 of $x$ (clearing a bit means making it 0).

- Multiply $x$ by 8.

- Extract bits 4 through 7 of $x$ (i.e., if $x$ is $b_7b_6b_5b_4b_3b_2b_1b_0$, your expression should evaluate to 0000$b_7b_6b_5b_4$).

- Round $x$ down to the nearest smaller multiple of 4 (e.g., 9 should become 8, 4 should remain 4, and $-9$ should become $-12$).
Putting that 173 pre-req to use

Recall that in 2’s complement, $-x \equiv \sim x + 1$. Using this, prove that $-x \equiv \sim (x - 1)$. Then use this new identity to explain what the following expression accomplishes: $(x + 3) \& -4$. *Hint:* Try it out on a few numbers to see what’s happening.