Recall \{\text{NOT, AND, OR}\} logically complete

\[ f(A, B, C) = A' + BC \]

\[ g(A, B, C) = (A + B) (A' + C) \]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{diagram.png}
\end{figure}
Question: Are there any other sets of logically complete function?

NAND function

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>(AB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Is this logically complete? To show this any set known to be logically complete \{ NOT, AND, OR \} is logically complete

(i) NOT using NAND √

\[ \neg \neg A = A \]
\[(A \cdot A)' = A' \quad A \cdot A = A\]

(ii) AND using NAND

\[A \quad (A \cdot B)' \quad A \cdot B\]

(iii) OR using NAND

\[A \quad (A' \cdot B')' \quad (A' \cdot B')' \rightarrow A + B\]
This proves that NAND is logically complete.

Second: NOR gate. Logically complete?

(i) NOT using NOR

\[ A \rightarrow \overline{A} \]

(ii) AND using NOR

\[ (A' + B')' \text{ using } \text{De Morgan's law } = A \cdot B \]

\[ A \rightarrow \overline{A} \rightarrow AB \]
(iii) OR using NOR

\[(A + B)\]' = A + B\]

Proving that NOR is also logically complete.

SOP: expression using NAND gates.

\[f(x, y, z) = xy + x'y'z\]

Using AND-OR

assum complemented inputs are available
Equivalently

AND

OR

\[ f \]
Likewise, POS is implemented using OR-AND network. (assume complemented inputs available)

\[ \text{POS } g(x, y, z) = (x + y') (x' + y) (y + z) \]
Use NOR

\[
\begin{align*}
&x' \\
y' \\
&y \\
&z
\end{align*}
\]

OR

AND

\[
\begin{align*}
x' \\
y'
\end{align*}
\]
2-level network

* A circuit is 2-level if there are at most 2 gates between any input and the output

* Every function can be represented in SOP form, thus it can be implemented with a 2-level AND-OR network or a 2-level NAND-NAND network
Every function can be represented in POS form, thus it can be implemented with a 2-level OR-AND network or a 2-level NOR-NOR network.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A \oplus B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

**Exclusive OR**

**Controlled inverter**