Lecture 2: Representation using bits; Unsigned and signed integers

Binary digits, 0/1 bits

Convention

0 - absence of voltage
1 - presence of voltage

Representation using bits

- How many things can we represent using k bits?

\[ 2^k \]

- 01
- 10
- 11
- 00

2 bits - 4 patterns

\[ 2 \times 2 = 2^2 = 4 \]
$2 \times 2 = 2^2 = 4$

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$k$ bits $= 2 \times 2 \times \ldots \times 2 \quad \text{\(k\) times} = 2^k$

**Inverse question**

How many bits are needed to represent $m$ things?

$m = 2^k$

$log_2 m = k$

$k \geq log_2 m$

(integer at least $log_2 m$)

0 16

$k \geq log_2 16 = 4$

4 bits

@ 30 $\rightarrow$ 5 bits

$log_2 30$.

16 32
Unsigned Integers: 0, 1, 2, …

1. Get the number
   a. If number is odd subtract 1
   b. If number is even subtract 0

2. Divide the resulting number by 2
3. Repeat until the resulting number is 0

LSB → MSB

\[ 68 \rightarrow 34 \rightarrow 17 \rightarrow 8 \rightarrow 4 \rightarrow 2 \rightarrow 1 \rightarrow 0 \]

LSB - least significant bit
MSB - most significant bit

\[ 68_{10} = 1000100_2 \]
\[ \begin{align*}
5_{(10)} & \rightarrow 101_{(2)} \\
55_{(10)} & = 5 \times 10^1 + 5 \times 10^0 \\
& = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\
k \text{ bits} & \\
\text{Range} & = [0, 2^k - 1] \\
& = [0, 2^8 - 1] \\
-3 & \rightarrow 11 \\
\text{Signed - Magnitude Representation} &
\end{align*} \]
Signed - Magnitude

3 bits

+0 0 0 0
+1 0 0 1
+2 0 1 0
+3 0 1 1

-0 1 0 0
-1 1 0 1
-2 1 1 0
-3 1 1 1

2's Complement Representation: Leading 0 if no. is +ve

Leading 1 if no. is -ve

a) Decimal → Binary

→ If no. is +ve do nothing (follow unsigned)

→ If no. is -ve:
  . Take its positive part (magnitude)
- Flip bits
- Add 1

\[
\begin{align*}
e.g.: (1) & \quad 5 = 0101_2 \\
& \quad \overline{5} = ? \\
5 & \rightarrow \begin{cases} 
5 \\
\text{flip} \\
+1 \\
011 \\
\text{negative}
\end{cases}
\end{align*}
\]

\[
\begin{align*}
& \quad 101 \\
& \quad \overline{1} \\
& \quad \overline{0} \\
& \quad \overline{0}
\end{align*}
\]

1011 (Unsigned?)
Signed?

Converting Binary to Decimal
Converting Binary to Decimal

Unsigned:

\[ 101₂ = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 5 \]

Signed (2's Complement)

- Note the sign
- Flip bits
- Add 1
- Convert to decimal

Aside

4 bits unsigned \(5₁₀\)

4 bits 2's Complement

\[ 0101 \]

\[ 0101 = 5₁₀ \]
Unsigned 5 = 6 bits

Signed (2's complement) 6 bits

\[-5 = \overline{111011} \] Sign Extension

Range 2's complement

\([-2^{k-1}, 2^{k-1}-1]\]

0 through \(2^{k-1}-1\)

\[-2^{k-1}\]