REPRESENTATION USING BITS

Basic concepts

* Devices operate by reacting to the presence of voltages

* Convention:
  0 denotes no voltage present \( \rightarrow \) Binary digit
  1 denotes voltage present \( \rightarrow \) (bit)
* We'll use bits to represent numbers:
  with $K$ bits we can represent $2^K$ numbers.

E.g. 2 bits:

Question: how many bits do we need to represent $M$ things?
Binary representation of integers

Decimal numbers use base 10 notation:

Binary numbers use base 2 notation:

To avoid confusion: \(101_2 = 5_{10}\)
With $k$ bits we can represent $\underline{2^k}$ numbers:

$0, 1, 2, \ldots, \underline{2^k}$

Finite capacity of binary representation:
Decimal to binary conversion

Given a number $m$:

1) Subtract 0 or 1 to make it even
2) Divide by 2 the even number you get
3) Repeat until the even number you get is zero

Example:
Question: How do we represent negative numbers?

**Signed magnitude**

* Idea: With $k$ bits you have $2^k$ numbers
  Let half of the numbers be positive
  Let other half be negative

* How to convey sign of number?
  Use leading digit: $0 \rightarrow$ positive \((\text{convention})\)
  $1 \rightarrow$ negative
Examples:
Z's complement

* Positive numbers are represented as we saw before for unsigned representation with Most Significant Bit (MSB) set to 0.

* To represent a negative number \(-A\):
  1) First represent the positive number \(+A\).
  2) Flip all its bits \(\rightarrow \text{complement}(A)\).
  3) Add 1 to the result:
     \[-A = \text{complement}(A) + 1\]
* Example: given $7_{10} = 00111_{2}$ find $-7_{10}$
Question: with \( k \) bits, how many positive numbers can be represented?

Question: with \( k \) bits, how many negative numbers can be represented?

* 2's complement allow us to represent with \( k \) bits numbers in the range \(-2^{k-1}\) to \(2^{k-1}-1\)

Binary to decimal conversion using 2's complement

Check leading bit (MSB):

1) If MSB = 0, positive number: compute right away

2) If MSB = 1, negative number: flip all bits and add 1 (2's complement)

Example:
Example: