ECE 120 Second Midterm Exam  
Spring 2016  
Tuesday, March 15, 2016

- Name: ___________________  
- NetID: ________________

Discussion Section:

<table>
<thead>
<tr>
<th>Time</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>9:00 AM</td>
<td>[ ] AB1</td>
</tr>
<tr>
<td>10:00 AM</td>
<td>[ ] AB2</td>
</tr>
<tr>
<td>11:00 AM</td>
<td>[ ] AB3</td>
</tr>
<tr>
<td>12:00 PM</td>
<td>[ ] AB4</td>
</tr>
<tr>
<td>1:00 PM</td>
<td>[ ] AB5 [ ] ABA</td>
</tr>
<tr>
<td>2:00 PM</td>
<td>[ ] AB6</td>
</tr>
<tr>
<td>3:00 PM</td>
<td>[ ] AB7 [ ] ABB</td>
</tr>
<tr>
<td>4:00 PM</td>
<td>[ ] AB8 [ ] ABC</td>
</tr>
<tr>
<td>5:00 PM</td>
<td>[ ] AB9 [ ] ABD</td>
</tr>
</tbody>
</table>

- Be sure that your exam booklet has 8 pages.  
- Write your name, netid and check discussion section on the title page.  
- Do not tear the exam booklet apart.  
- Use backs of pages for scratch work if needed.  
- This is a closed book exam. You may not use a calculator.  
- You are allowed one handwritten 8.5 x 11” sheet of notes (both sides).  
- Absolutely no interaction between students is allowed.  
- Clearly indicate any assumptions that you make.  
- The questions are not weighted equally. Budget your time accordingly.  
- Show your work.

Problem 1    20 points    __________
Problem 2    19 points    __________
Problem 3    18 points    __________
Problem 4    14 points    __________
Problem 5    17 points    __________
Problem 6    12 points    __________

Total        100 points    __________
Problem 1 (20 points): CMOS and Boolean properties

1. (5 points) Circle the correct choice for each statement. The inputs are inverted.

   a. The output Z equals:

   \[ P \text{ AND } Q \quad P \text{ OR } Q \quad P \text{ NOR } Q \quad P \text{ XOR } Q \quad P \text{ XNOR } Q \]

   b. This example best illustrates the Boolean property:

   DeMorgan’s    Absorption    No-Name    Consensus

2. (15 points) Let \( G(x,y,z) \) and \( H(x,y,z) \) be the 3-variable functions whose K-maps are given below.

\[
\begin{array}{c|ccc}
G(x,y,z) & yz & & \\
\hline
x & 00 & 01 & 11 & 10 \\
0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 0
\end{array}
\quad
\begin{array}{c|ccc}
H(x,y,z) & yz & & \\
\hline
x & 00 & 01 & 11 & 10 \\
0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0
\end{array}
\]

   a. Express \( H(x,y,z) \) in canonical POS form

   i. Using the variables \( x, y, z \):

   \[ H(x,y,z) = \]

   ii. Using the maxterm \( M_i \) notation:

   \[ H(x,y,z) = \]

   b. Using your expression for \( H \) from part a.i), give the exact dual of \( H \):

   dual of \( H(x,y,z) = \)

   c. Complete the K-map (below) for function \( F \), so that \( F + G = H \). You must use don’t cares wherever possible.

\[
\begin{array}{c|ccc}
F(x,y,z) & yz & & \\
\hline
x & 00 & 01 & 11 & 10 \\
0 & \text{[ ] } & \text{[ ] } & \text{[ ] } & \text{[ ] } \\
1 & \text{[ ] } & \text{[ ] } & \text{[ ] } & \text{[ ] }
\end{array}
\]
Problem 2 (19 points): Sequential logic

1. (10 points) Consider the sequential feedback circuit shown below.

   M → Q
   N → Q

   a. Complete the next-state table for this circuit

<table>
<thead>
<tr>
<th>M</th>
<th>N</th>
<th>Q⁺</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Forbidden</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

   b. Express the next state Q⁺ as a function of M, N, and Q in SOP form.

   Q⁺ =

2. (9 points) Consider a 3-bit shift register that has the following diagram:

   a. Determine the functionality of the register by completing the following table

<table>
<thead>
<tr>
<th>F₁</th>
<th>F₀</th>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Unused</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

   b. If the shift register initially stores Q₂Q₁Q₀=100 and Input=0, what is stored in the register after one clock pulse and

   F₁ F₀ = 01?

   F₁ F₀ = 10? (Assume again that 100 is stored before the operation.)

   F₁ F₀ = 11? (Assume again that 100 is stored before the operation.)
Problem 3 (18 points)

Consider the 4-variable function \( f(w,x,y,z) \), with the following K-map (drawn twice).

1. Give a minimal SOP expression for \( f(w,x,y,z) \) and show the corresponding loops on the left map.

   \[
   \text{min SOP: } \quad \text{loops on left map.}
   \]

2. Give a minimal POS expression for \( f(w,x,y,z) \) and show the corresponding loops on the right map.

   \[
   \text{min POS: } \quad \text{loops on right map.}
   \]

3. Implement \( f \) using only a 4:1 multiplexer (with select inputs \( S_1S_0 = wx \)) and one NAND gate. Complemented inputs are not available.
Problem 4 (14 points)

In this problem you will complete the design of the circuit shown below, which compares two 4-bit unsigned binary numbers A=a₃a₂a₁a₀ and B=b₃b₂b₁b₀ and outputs

\[ f = \begin{cases} 
  x \text{ NAND } y & \text{if } A < B \\
  x' \text{ OR } y & \text{if } A \geq B 
\end{cases} \]

1. (8 points) Design cell C so that the comparator portion of the above circuit operates correctly and outputs

\[ c_4 = \begin{cases} 
  0 & \text{if } A < B \\
  1 & \text{if } A \geq B 
\end{cases} \]

   a. Specify the input \( c_0 \).

\[ c_0 = \]

   b. Express \( c_{i+1} \) in terms of \( c_i, a_i, b_i \).

\[ c_{i+1} = \]

2. (6 points) Design the network N by giving a Boolean expression for \( f \).

\[ f = \]
Problem 5 (17 points)

Shown below is an 8-bit arithmetic unit (AU) which operates on two 8-bit 2’s complement numbers A and B. Each network N computes a* and b*, where:

\[
a^* = k_1' k_0' + k_1 a
\]

\[
b^* = k_1 k_0' b' + k_0 b + k_1' k_0' a
\]

1. (4 points) Give a 2-level NAND gate implementation of a*. Assume complemented inputs are available.

2. (13 points) Complete the table below.
   a. Give the values for a*, b*, c_0
   b. Specify the operation performed. Express your answer as an arithmetic function (PLUS/MINUS) of A and B (e.g., “a plus the complement of b” is not an appropriate response).

<table>
<thead>
<tr>
<th>k_1</th>
<th>k_0</th>
<th>a*</th>
<th>b*</th>
<th>c_0</th>
<th>Operation performed as a function of A and B (e.g. A PLUS/MINUS B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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</table>
Problem 6 (12 points): Finite State Machines

The circuit below is a 2-bit register that shifts right with serial input of 0 when \( K=0 \) and parallel loads with inputs of 1 when \( K=1 \).

1. (6 points) Complete the state transition table for the circuit.

<table>
<thead>
<tr>
<th>( S_1 )</th>
<th>( S_0 )</th>
<th>( K )</th>
<th>( S'_1 )</th>
<th>( S'_0 )</th>
<th>( Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
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</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. (6 points) Complete the state transition diagram for the circuit.

FSM notation:

\[
\begin{array}{c}
S_1 S_0/Z \\
\end{array}
\]

\[
\begin{array}{c}
00/\_
\\
01/\_
\\
11/\_
\\
10/\_
\end{array}
\]
**Boolean algebra properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation 1</th>
<th>Equation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Commutativity</strong></td>
<td>$x \cdot y = y \cdot x$</td>
<td>$x + y = y + x$</td>
</tr>
<tr>
<td><strong>Associativity</strong></td>
<td>$(x \cdot y) \cdot z = x \cdot (y \cdot z)$</td>
<td>$(x + y) + z = x + (y + z)$</td>
</tr>
<tr>
<td><strong>Distributivity</strong></td>
<td>$x \cdot (y + z) = x \cdot y + x \cdot z$</td>
<td>$x + y \cdot z = (x + y) \cdot (x + z)$</td>
</tr>
<tr>
<td><strong>Idempotence</strong></td>
<td>$x \cdot x = x$</td>
<td>$x + x = x$</td>
</tr>
<tr>
<td><strong>Identity</strong></td>
<td>$x \cdot 1 = x$</td>
<td>$x + 0 = x$</td>
</tr>
<tr>
<td><strong>Null</strong></td>
<td>$x \cdot 0 = 0$</td>
<td>$x + 1 = 1$</td>
</tr>
<tr>
<td><strong>Complementarity</strong></td>
<td>$x \cdot x' = 0$</td>
<td>$x + x' = 1$</td>
</tr>
<tr>
<td><strong>Involution</strong></td>
<td>$(x')' = x$</td>
<td></td>
</tr>
<tr>
<td><strong>DeMorgan’s</strong></td>
<td>$(x \cdot y)' = x' + y'$</td>
<td>$(x + y)' = x' \cdot y'$</td>
</tr>
<tr>
<td><strong>Absorption</strong></td>
<td>$x \cdot (x + y) = x$</td>
<td>$x + x \cdot y = x$</td>
</tr>
<tr>
<td><strong>No-Name</strong></td>
<td>$x \cdot (x' + y) = x \cdot y$</td>
<td>$x + x' \cdot y = x + y$</td>
</tr>
<tr>
<td><strong>Consensus</strong></td>
<td>$(x+y) \cdot (y+z) \cdot (x'+z) =$</td>
<td>$x \cdot y + y \cdot z + x' \cdot z =$</td>
</tr>
<tr>
<td></td>
<td>$(x+y) \cdot (x'+z)$</td>
<td>$x \cdot y + x' \cdot z$</td>
</tr>
</tbody>
</table>

Feel free to tear this page off and use it as scratch paper.