Communication/Computation Ratio

Notation: \( N = \) problem size (\( N \) by \( N \) matrix)
\( m = \) problem size = \( kN^2 \)
\( P = \) number of processors
\( a, a', b, b' = \) constants

1). Row Decomposition of Jacobi Relaxation Problem

Each processor has \( a\frac{N^2}{P} \) computation amount

\[
T_{\text{compute}} = a' \frac{N^2}{P} = \frac{am}{P}
\]

\[
T_{\text{communication}} = b'N = b\sqrt{m}
\]

\[
T_{\text{parallel}} = \frac{am}{P} + b\sqrt{m}
\]

\[
T_{\text{seq}} = am
\]

\[
\text{Speedup} = \frac{\text{seq time}}{\text{parallel time}} = \frac{am}{\frac{am}{P} + b\sqrt{m}} = \frac{amP}{am + b\sqrt{m}} = \frac{amP}{a\sqrt{m} + bP}
\]

\[
\eta = \frac{\text{speedup}}{\# \text{processors}} = \frac{a\sqrt{m}}{a\sqrt{m} + bP}
\]

\[
\gamma = \frac{\text{Communication}}{\text{Computation}} = \frac{b\sqrt{m}P}{am} = \frac{bP}{a\sqrt{m}}
\]

So, \( \eta = \frac{a\sqrt{m}}{a\sqrt{m} + bP} = \frac{1}{1 + \gamma} \)

If we double the number of processors \( \eta \) decreases, thus we need to quadruple \( m \) to get same \( \eta \)
Thus, \( Isoeff_{\text{row}} = m^2 \)

2). Block Decomposition of Jacobi Relaxation Problem

\[
T_{\text{compute}} = \frac{am}{P} \text{ (same)}
\]

\[
T_{\text{communication}} = b\sqrt{m}
\]

\[
\gamma = \frac{b\sqrt{m}P}{a\sqrt{m}} = \frac{b\sqrt{P}}{a\sqrt{m}}
\]

So, \( Isoeff_{\text{block}} = \text{linear} \)