Example from class 1/25/11

This example was rushed today. Here are the details.

\[ \int \sqrt{4-x^2} \, dx \]

Let \( x = 2 \sin \theta \), \( -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \)

\[ dx = 2 \cos \theta \, d\theta \]

\[ \sqrt{4-x^2} = \sqrt{4-4 \sin^2 \theta} = \sqrt{4 \cos^2 \theta} = 2 \cos \theta \]

So,

\[ \int \sqrt{4-x^2} \, dx = \int 2 \cos \theta \cdot 2 \cos \theta \, d\theta \]

\[ = \int 4 \cos^2 \theta \, d\theta = \int 2(1 + \cos 2\theta) \, d\theta \]

(by trig. identity)

\[ = 2\theta + \sin 2\theta + C \]

(by trig. identity)

\[ = 2\theta + 2 \sin \theta \cos \theta + C \]

\[ = 2 \arcsin \left( \frac{x}{2} \right) + x \cdot \frac{\sqrt{4-x^2}}{2} + C \]

(Plug in the values from the box above)