Example

Lamina of density $\rho$ by $\Delta$ is isosceles $\Delta$ of height $L$, base $2L$ as shown.
Find centroid and moments about $x, y$ axes.

By symmetry $\bar{x} = 0$.
So $M_x = \bar{y} \cdot \rho \cdot 0$ (not tendency to rotate about $y$-axis).

To compute $M_x$: take strip at $y$ as shown.

Length of strip = $2(L-y)$ by similar $\Delta$.
Area of strip = $2(L-y)dy$.
Mass of strip = $2\rho(L-y)dy$.

Moment of strip about $x$-axis: $dM_x = 2 \rho y(L-y)dy$.

Moment: $M_x = \int_0^L 2 \rho y(L-y)dy = 2 \rho \left[ \frac{L^2y}{2} - \frac{y^3}{3} \right]_0^L$

$= 2 \rho \left( \frac{L^3}{2} - \frac{L^3}{3} \right) = \frac{L^3 \rho}{3}$

$\bar{x} = \frac{1}{2} L$

$\bar{y} = \frac{1}{3} L$

$M = \text{area} \cdot \text{density}$

$\frac{PL}{3} = \rho L \bar{y}$

$\bar{y} = \frac{L}{3}$