Solution to homework problem

\[ x^2 + y^2 = 1 \]

density \( \lambda \)

Note:
Can add moments

\[
\begin{align*}
\frac{m_1 y_1 + m_2 y_2}{m_2} + m_2 y_2
\end{align*}
\]

Find \( M_x, M_y \)

\[ \text{Mx and My for top} \]

Contribution of strip at \( x \)

\[
dM_y = (\text{dist. to x-axis})(\text{mass of strip})
\]

\[ = x \cdot 2\lambda \sqrt{1-x^2} \, dx \]

\[ M_y = \int_0^1 2\lambda x \sqrt{1-x^2} \, dx = \lambda \left[ \frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{\lambda}{6} \]

mass = \[ \int_0^1 \lambda \sqrt{1-x^2} \, dx 
\]

By symmetry, \( M_x = \frac{\lambda}{6} \) also.

\[ \text{Mx and My for bottom} \]

Contribution of strip at \( x \)

\[
dM_y = (\text{dist. to x-axis})(\text{mass of strip})
\]

\[ = x \cdot 2\lambda (1-x) \, dx \]

\[ M_y = \int_0^1 2\lambda x (1-x) \, dx = \lambda \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{\lambda}{6} \]

mass = \[ \int_0^1 \lambda (1-x) \, dx \]

By symmetry, \( M_x = -\frac{\lambda}{6} \)

For combined region:

\[
M_x = \frac{\lambda}{3} - \frac{\lambda}{6} = \frac{\lambda}{6}
\]

\[
M_y = \frac{\lambda}{3} + \frac{\lambda}{6} = \frac{\lambda}{2}
\]

\[
M = \frac{1}{2} \pi + \frac{1}{2} \lambda
\]

\[ (\bar{x}, \bar{y}) = \left( \frac{M_y}{M}, \frac{M_x}{M} \right) \]

\[ = \left( \frac{\lambda}{3}, \frac{\lambda}{2} \right) \]