1. (3 points) Find \( \lim_{x \to \infty} (10e^{3x} + 8)^{5/x} \) 

\[
\lim_{x \to \infty} \ln \left( 10e^{3x} + 8 \right)^{5/x} = \lim_{x \to \infty} \frac{5}{x} \ln \left( 10e^{3x} + 8 \right)
\]

\[
= \lim_{x \to \infty} \frac{5 \ln (10e^{3x} + 8)}{x} \times \frac{1}{\lim_{x \to \infty} \frac{1}{10e^{3x} + 8}} \times \frac{1}{\lim_{x \to \infty} \frac{30e^{3x}}{10e^{3x} + 8}}
\]

\[
= \lim_{x \to \infty} \frac{5(30e^{3x})}{10e^{3x} + 8} \times \frac{1}{\lim_{x \to \infty} \frac{90e^{3x}}{30e^{3x}}}
\]

\[
= \lim_{x \to \infty} 15 = 15
\]

\[
15 = \ln(L)
\]

\[
e^{15} = e^{\ln(L)}
\]

\[
L = e^{15}
\]
2. (4 points) Consider the following function, \( f(x) = \frac{x - 5}{e^x} \)

(a) Where is \( f(x) \) equal to 0 or undefined? Where is \( f(x) \) positive, negative?

\[
\begin{align*}
\text{\( f(x) \) is defined everywhere} \quad f(x) = 0 & \quad \text{when} \quad x = 5 \\
\frac{e^x}{x-5} = 0 \; & \; \; \text{\( f(x) \) is positive, negative} \quad f(x) > 0 \; \; \text{when} \; \; x > 5 \\
\frac{e^x}{x-5} = 0 \; & \; \; \text{\( f(x) \) is positive, negative} \quad f(x) < 0 \; \; \text{when} \; \; x < 5
\end{align*}
\]

(b) What are the horizontal and vertical asymptotes of \( f(x) \)?

\[
\begin{align*}
\text{No VA} & \quad \text{b/c} \; e^x \neq 0 \; \; \text{for all real} \; x \\
\lim_{x \to \infty} \frac{x-5}{e^x} & = \lim_{x \to \infty} \frac{1}{e^x} = 0 \\
\lim_{x \to -\infty} \frac{x-5}{e^x} & \rightarrow -\infty
\end{align*}
\]

There is a HA at \( y = 0 \) (off to the right)

(c) Where are the critical points of \( f(x) \)? Where is \( f(x) \) increasing, decreasing?

\[
\begin{align*}
f'(x) = \frac{e^x(1) - (x-5)e^x}{(e^x)^2} & = \frac{6-x}{e^x} \\
f'(x) = 0 \; \; \text{when} \; \; x = 6 \\
\text{\( x = 6 \) is a critical point} \\
(6, \frac{1}{e^6}) & \; \; \text{\( f(x) \) is increasing} \; \; \text{when} \; \; f'(x) > 0 \; \; \text{when} \; \; x < 6 \\
\text{\( f(x) \) is decreasing} \; \; \text{when} \; \; f'(x) < 0 \; \; \text{when} \; \; x > 6
\end{align*}
\]

(d) Where is \( f''(x) = 0? \) Where is \( f(x) \) concave up, concave down? What, if any, are the inflection points of \( f(x) \)?

\[
\begin{align*}
f''(x) = \frac{e^x(-1) - (6-x)e^x}{(e^x)^2} & = \frac{x-7}{e^x} \\
f''(x) = 0 \; \; \text{when} \; \; x = 7 \\
\text{\( f(x) \) is concave up \( (-\infty, 7) \) \( f(x) \) is concave down \( (7, \infty) \)
\end{align*}
\]

There is an inflection point at \( (7, \frac{7}{e^7}) \), where \( f(x) \) changed concavity
3. (3 points) A boat leaves a dock at 5:00 PM and travels due south at a speed of 5 km/h. Another boat has been heading due east at 10 km/h and reaches the same dock at 6:00 PM. At what time after 5:00 PM were the two boats closest together?

\[ X = 5t \]
\[ Y = 10t \]

\[ \text{distance} = D = \sqrt{x^2 + (10-y)^2} = \sqrt{25t^2 + 100(1-t)^2} \]

Minimize distance \( D \) by minimizing \( D^2 \)

\[ D^2 = 25t^2 + 100(1-t)^2 \]

\[ (D^2)' = 50t - 200(1-t)(-1) \]

\[ (D^2)' = 50t + 200(1-t)(-1) \]

\[ 50t - 200 + 200t = 0 \]
\[ 250t = 200 \]
\[ t = \frac{4}{5} \]

Since there is only 1 critical point, \( t = \frac{4}{5} \) is an absolute minimum

Distance is minimized \( \frac{4}{5} \) hour after 5 pm

\[ 5:48 \text{pm} \]