7.7: Approximate Integration

- You need to know the formulas for the left endpoint approximation, right endpoint approximation, midpoint rule, trapezoidal rule, and Simpson’s rule.
- There are pictures of what each of these rules looks like in Section 7.7 of the textbook.
- In each of the following formulas, we divide the interval \([a, b]\) into \(n\) subintervals of equal width. If \(i\) is some natural number from 1 to \(n\), we write the \(i\)th subinterval as \([x_{i-1}, x_i]\). The width of each subinterval, which we write as \(\Delta x\), is \(\frac{b-a}{n}\). So \(x_0 = a, x_1 = a + \Delta x, x_2 = a + 2\Delta x, \text{etc.}\) and \(x_n = b\).

- **Left endpoint approximation:** The left endpoint approximation to \(\int_a^b f(x) \, dx\) with \(n\) subintervals, \(L_n\), is:
  \[
  \int_a^b f(x) \, dx \approx L_n = \Delta x(f(x_0) + f(x_1) + \ldots + f(x_{n-1}))
  \]

- **Right endpoint approximation:** The right endpoint approximation to \(\int_a^b f(x) \, dx\) with \(n\) intervals, \(R_n\), is:
  \[
  \int_a^b f(x) \, dx \approx R_n = \Delta x(f(x_1) + f(x_2) + \ldots + f(x_n)).
  \]

- **Midpoint rule:** The midpoint approximation to \(\int_a^b f(x) \, dx\) with \(n\) intervals, \(M_n\), is:
  \[
  \int_a^b f(x) \, dx \approx M_n = \Delta x(f(\bar{x}_1) + f(\bar{x}_2) + \ldots + f(\bar{x}_n)).
  \]
  where \(\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)\), the midpoint of the \(i\)th subinterval \([x_{i-1}, x_i]\).

- **Trapezoidal rule:** The trapezoidal approximation to \(\int_a^b f(x) \, dx\) with \(n\) intervals, \(T_n\), is
  \[
  \int_a^b f(x) \, dx \approx T_n = \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n)).
  \]

- **Simpson’s rule:** If \(n\) is an even number, then Simpson’s rule for approximating \(\int_a^b f(x) \, dx\) with \(n\) intervals, \(S_n\), is
  \[
  \int_a^b f(x) \, dx \approx S_n = \frac{\Delta x}{3}(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)).
  \]

- If \(f\) is decreasing on \((a, b)\):
  - \(R_n < \int_a^b f(x) \, dx < L_n\)
- If \(f\) is increasing on \((a, b)\):
  - \(L_n < \int_a^b f(x) \, dx < R_n\)
- If \(f\) is concave up on \((a, b)\):
  - \(M_n < \int_a^b f(x) \, dx < T_n\)
- If \(f\) is concave down on \((a, b)\):
  - \(T_n < \int_a^b f(x) \, dx < M_n\)
8.1: Arc length

- General formula for arc length: \( L = \int ds \), where

\[
ds = \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \quad \text{or} \quad ds = \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy.
\]

- If the curve we want to find the arc length of is described by \( y = f(x) \), \( a \leq x \leq b \), it may be better to use the first formula above. If the curve is described by \( x = g(y) \), \( c \leq y \leq d \), it may be better to use the second formula. (Of course, if you are asked to write the integral in terms of \( x \), then you should use the first formula. If you are asked to write the integral in terms of \( y \), then you should use the second formula.)

- The bounds on the arc length integral should be the bounds on the curve in \( x \) or \( y \).

8.2: Surface area

- General formula for surface area obtained by rotating a curve around a line: \( SA = \int 2\pi R \, ds \). In this formula:

  - \( R \) is the distance from the curve to the line we rotate around. If we rotate around the \( y \)-axis, then \( R = x \), and if we rotate around the \( x \)-axis, then \( R = y \).

  - \( ds \) is either \( \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx \) or \( \sqrt{1 + \left( \frac{dx}{dy} \right)^2} \, dy \).

The bounds for the integral should be the bounds on \( x \) or \( y \) that describe the curve we’re rotating.

- Note that \( R \) depends only on the line that we rotate around.

- Important: When writing down the integral for surface area, your integral should be all in terms of \( x \) or all in terms of \( y \). If your formula for \( ds \) involves \( dx \), you should make sure that \( R \) is a function of \( x \). If your formula for \( ds \) involves \( dy \), you should make sure that \( R \) is a function of \( y \).

8.3: Applications to physics and engineering

**Hydrostatic force**

- General outline of the steps to find the hydrostatic force on an underwater shape:

  - Set up a ruler. Pick a point to be \( y = 0 \) and decide which direction \( y \) is increasing in.

  - Divide the underwater shape into horizontal strips of height \( dy \). Find the area of a strip at height \( y \).

  - The force from the water on one of these strips is

\[
dF = (\rho g d(y))dA.
\]

In this formula:

- \( \rho \) and \( g \) are constants; \( \rho \) is the density of water and \( g \) is the acceleration due to gravity.

- \( d(y) \) means the depth of a strip at height \( y \) under the surface of the water. This should be some expression in terms of \( y \).

- \( dA \) means the area of a strip at height \( y \).

- \( \rho g d(y) \) is the pressure from the water on the strip, and the force on the strip is pressure times area.

The force on the entire underwater shape is \( F = \int dF \). The bounds on the integral should be the bounds on the part of the shape that is underwater.

- The reason we divide the shape up into horizontal strips (and not vertical strips) is that pressure in a fluid depends only on depth.

- It is often helpful to set up your coordinate system in such a way that the shape you are considering has a nice equation. For example, when finding the hydrostatic force on a circle (or semicircle), it can be helpful to set up your coordinate system so that the origin is at the center of the circle.
Moments and centers of mass

- Suppose we have a lamina with area density \( \lambda \) and the lamina is described as the area between two curves \( y = f(x) \) and \( y = g(x) \), \( a \leq x \leq b \), where \( f(x) \leq g(x) \). Here is a general outline of the steps to find the moments \( M_x \) and \( M_y \) of the lamina about the \( x- \) and \( y- \) axes, respectively.
  - Divide the lamina into vertical strips of width \( dx \).
  - Find the moment of one of these strips about the \( x- \) axis, \( dM_x \). In this case, the formula for \( dM_x \) is:
    \[
    dM_x = \lambda \left( (g(x) + f(x))(g(x) - f(x)) \right) dx
    = \lambda \left( (g(x))^2 - (f(x))^2 \right) dx
    \]
    In this formula:
    * \( \frac{1}{2}(g(x) + f(x)) \) gives the location of the center of one of these strips above or below the \( x- \) axis.
    * \( (g(x) - f(x))dx \) gives the area of one of these strips, and \( \lambda(g(x) - f(x))dx \) gives the mass of one of these strips.
    Then \( M_x = \int_a^b \lambda \left( (g(x))^2 - (f(x))^2 \right) dx \).
  - If \( f(x) = 0 \) (which is often the case) and the lamina is just the area below the graph of the function \( g(x) \), \( a \leq x \leq b \), then this simplifies to
    \[
    M_x = \int_a^b \frac{\lambda}{2}(g(x))^2 \ dx.
    \]
  - Find the moment of one of these strips about the \( y- \) axis, \( dM_y \). In this case, the formula for \( dM_y \) is:
    \[
    dM_y = \lambda x(g(x) - f(x)) \ dx.
    \]
    In this formula:
    * \( x \) gives the location of the center of one of these strips to the right or left of the \( y- \) axis.
    * \( g(x) - f(x)dx \) gives the area of one of these strips, and \( \lambda(x(g(x) - f(x))dx \) gives the mass of one of these strips.
    Then \( M_y = \int_a^b dM_y = \int_a^b \lambda x(g(x) - f(x)) \ dx \).
  - If \( f(x) = 0 \), then this simplifies to
    \[
    M_y = \int_a^b \lambda xg(x) \ dx.
    \]

- Suppose we have a lamina with area density \( \lambda \) and the lamina is described as the area between two curves \( x = f(y) \) and \( x = g(y) \), \( c \leq x \leq d \), where \( f(y) \) is to the left of \( g(y) \). Here is a general outline of the steps to find the moments \( M_x \) and \( M_y \) of the lamina about the \( x- \) and \( y- \) axes, respectively.
  - Divide the lamina into horizontal strips of width \( dy \).
  - Find the moment of one of these strips about the \( x- \) axis, \( dM_x \). In this case, the formula for \( dM_x \) is:
    \[
    dM_x = \lambda y(g(y) - f(y)) \ dy.
    \]
    In this formula:
    * \( y \) gives the location of the center of one of these strips above or below the \( x- \) axis.
    * \( (g(y) - f(y))dy \) gives the area of one of these strips, and \( \lambda(y(g(y) - f(y))dy \) gives the mass of one of these strips.
    Then \( M_x = \int_c^d dM_x = \int_c^d \lambda y(g(y) - f(y)) \ dy \).
– If \( f(y) = 0 \), so that the lamina is just the area between \( g(y) \) and the \( y \)-axis, \( c \leq y \leq d \), then this simplifies to

\[
M_x = \int_c^d \lambda y g(y) \, dy.
\]

– Find the moment of one of these strips about the \( y \)-axis, \( dM_y \). In this case, the formula for \( dM_y \) is:

\[
dM_y = \frac{\lambda}{2} (g(y) + f(y))(g(y) - f(y)) \, dy
\]

\[
= \frac{\lambda}{2} ((g(y))^2 - (f(y))^2) \, dy.
\]

In this formula:

* \( \frac{1}{2}(g(y) + f(y)) \) gives the location of the center of one of these strips to the right or left of the \( y \)-axis.

* \( (g(y) - f(y))dy \) gives the area of one of these strips, and \( \lambda(g(y) - f(y))dy \) gives the mass of one of these strips.

Then \( M_y = \int_c^d dM_y = \int_c^d \frac{\lambda}{2} ((g(y))^2 - (f(y))^2) \, dy \).

– If \( f(y) = 0 \), then this simplifies to

\[
M_y = \int_c^d \frac{\lambda}{2} (g(y))^2 \, dy.
\]

• To find the centroid (center of mass) of a shape:
  – Calculate \( M_x \) and \( M_y \) as above.
  – Find the area \( A \) of the entire lamina. The mass \( m \) of the entire lamina is then \( \lambda A \).
  – Let \( \bar{x} \) be the \( x \)-coordinate of the center of mass. Let \( \bar{y} \) be the \( y \)-coordinate of the center of mass. Then

\[
(\bar{x}, \bar{y}) = \left( \frac{M_y}{m}, \frac{M_x}{m} \right).
\]

• Rather than memorizing the formulas above, it is easier to think about the general idea behind how we find moment of a shape about the \( x \)- or \( y \)-axis. First we divide the lamina up into thin horizontal or vertical strips. Then the moment of one of these strips about an axis is given by:

\[
\text{(location of center of strip relative to \( x \)- or \( y \)-axis)} \ast \text{(mass of strip)}.
\]

After we find the moment of a strip about the \( x \)- or \( y \)-axis, we then integrate to get the moment of the whole shape about the \( x \)- or \( y \)-axis.

• A related note: You don’t have to follow the above steps when calculating the moment. For example, if your lamina is described as the region under the graph of a function \( y = f(x) \), \( a \leq x \leq b \), you can divide the lamina up into horizontal strips instead of vertical strips. Just be careful when setting up the integral; think about how to find the moment of a strip about the \( x \)- or \( y \)-axis like in the previous bullet point.

• If the lamina is symmetric about a line, then the centroid of the lamina lies on the line. In particular, if the lamina is symmetric about the \( y \)-axis, then \( \bar{x} = 0 \). If the lamina is symmetric about the \( x \)-axis, then \( \bar{y} = 0 \).

### 11.1: Sequences

• A sequence is a list of numbers in a definite order.

• If \( L \) is a real number, saying that \( \lim_{n \to \infty} a_n = L \) means that we can make the terms of the sequence \( \{a_n\} \) as close to \( L \) as we want if we take \( n \) to be large enough.

• Saying that \( \lim_{n \to \infty} a_n = \infty \) means that we can make the terms of the sequence \( \{a_n\} \) as big as we want if we take \( n \) to be large enough.
• The book has more precise definitions of the limit of a sequence in Section 11.1.

• If \( \lim_{n \to \infty} a_n \) is a real number, we say that the sequence \( \{a_n\} \) converges. Otherwise (if the limit is \( \pm \infty \) or the limit does not exist), we say that the sequence diverges.

• **Theorem** (3, Section 11.1): If \( \lim_{x \to \infty} f(x) = L \) and \( f(n) = a_n \) when \( n \) is a natural number, then \( \lim_{n \to \infty} a_n = L \). This tells us that if we want to find the limit of a sequence, we can use tricks for finding the limit as \( x \to \infty \) of a function \( f(x) \). For example, we can use L’Hopital’s rule to show that \( \lim_{x \to \infty} f(x) = L \), and then conclude that \( \lim_{n \to \infty} a_n = L \).

• **Squeeze theorem for sequences**: If \( a_n \leq b_n \leq c_n \) for \( n \geq N \) and \( \lim_{n \to \infty} a_n = \lim_{n \to \infty} c_n = L \), then \( \lim_{n \to \infty} b_n = L \).

• **Theorem** (6, Section 11.1): If \( \lim_{n \to \infty} |a_n| = 0 \), then \( \lim_{n \to \infty} a_n = 0 \).

• **Theorem** (7, Section 11.1): If \( \lim_{n \to \infty} a_n = L \) and the function \( f \) is continuous at \( L \), then

\[
\lim_{n \to \infty} f(a_n) = f(\lim_{n \to \infty} a_n) = f(L).
\]

• A sequence \( \{a_n\} \) is increasing if \( a_n < a_{n+1} \) for all \( n \). A sequence \( \{a_n\} \) is decreasing if \( a_n > a_{n+1} \) for all \( n \). A sequence is monotone if it is increasing or decreasing.

• You can check if a sequence \( \{a_n\} \) is increasing or decreasing by taking a function \( f(x) \) such that \( f(n) = a_n \) for all \( n \), and then checking if \( f'(x) > 0 \) or if \( f'(x) < 0 \). Sometimes this is not necessary. For example, since \( n < n+1 \) for all \( n \), \( \frac{1}{n+1} > \frac{1}{n} \) for all \( n \), and so \( \{ \frac{1}{n} \}_{n=1}^{\infty} \) is a decreasing sequence.

• A sequence \( \{a_n\} \) is bounded above if there is a number \( M \) such that \( a_n \leq M \) for all \( n \). A sequence is bounded below if there is a number \( m \) such that \( m \leq a_n \) for all \( n \).

• **Monotone convergence theorem**:

  - If \( \{a_n\} \) is increasing and bounded above, then \( \{a_n\} \) converges.
  - If \( \{a_n\} \) is decreasing and bounded below, then \( \{a_n\} \) converges.

11.2: **Series**

• A series is an infinite sum, \( \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots \).

• If we have a series \( \sum_{n=1}^{\infty} a_n \), let \( s_1 = a_1 \), \( s_2 = a_1 + a_2 \), \( s_3 = a_1 + a_2 + a_3 \), and so on. In general, let \( s_n = a_1 + \ldots + a_n \). \( s_n \) is called the \( n \)th partial sum and is the sum of the terms in the series up through \( a_n \).

• Fact: If \( n > 1 \), \( a_n = s_n - s_{n-1} \).

• If \( \lim_{n \to \infty} s_n \) is a real number \( s \), then we say that the series \( \sum_{n=1}^{\infty} a_n \) converges and we set \( \sum_{n=1}^{\infty} a_n = s \).

  If \( \lim_{n \to \infty} s_n \) is \( \pm \infty \) or does not exist, then we say that the series diverges.

• **Important note**: To say if a series converges or diverges, we look at the limit of the sequence of partial sums \( s_1, s_2, s_3, \ldots \). **Not** the limit of the sequence of terms of the series \( (a_1, a_2, a_3, \ldots) \).

• **Geometric series**: This is a special kind of series, a series of the form

\[
\sum_{n=0}^{\infty} ar^n = \sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + ar^3 + \ldots.
\]
In a geometric series, each term in the series is obtained by multiplying the previous term by the same number \( r \). If \( |r| < 1 \), then the geometric series converges and
\[
\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1 - r}.
\]
If \( |r| \geq 1 \), then the geometric series diverges.

- **Theorem** (6, Section 11.2): If the series \( \sum a_n \) is convergent, then \( \lim_{n \to \infty} a_n = 0 \). From this theorem we get the following important test for determining if a series diverges.

- **Test for Divergence** (Theorem 7, Section 11.2): If \( \lim_{n \to \infty} a_n \neq 0 \), then the series \( \sum a_n \) is divergent. In this theorem, \( \lim_{n \to \infty} a_n \neq 0 \) includes the cases where the limit is \( \pm \infty \) or when the limit does not exist.

- **Warning**: If \( \lim_{n \to \infty} a_n = 0 \), the series \( \sum a_n \) may converge or diverge. The Test for Divergence doesn’t apply in this case. In other words, we can’t conclude anything about the convergence of the series \( \sum a_n \) if \( \lim_{n \to \infty} a_n = 0 \). (Look at \( \sum_{n=1}^{\infty} \frac{1}{n} \), we have \( \lim_{n \to \infty} \frac{1}{n} = 0 \) but \( \sum_{n=1}^{\infty} \frac{1}{n} \) diverges.)

- Adding a finite number of terms to a series doesn’t affect its convergence or divergence. Similarly, multiplying a series by a nonzero constant doesn’t affect the convergence or divergence.

### 11.3: The Integral Test

- **The Integral Test**: Suppose \( f \) is a function that is continuous, positive, and decreasing on \([k, \infty)\) such that \( f(n) = a_n \). Then:
  - If \( \int_{k}^{\infty} f(x) \, dx \) converges, so does \( \sum_{n=k}^{\infty} a_n \).
  - If \( \int_{k}^{\infty} f(x) \, dx \) diverges, so does \( \sum_{n=k}^{\infty} a_n \).

- When trying to determine if \( \int_{k}^{\infty} f(x) \, dx \) converges, it is often helpful to use the Comparison Theorem for improper integrals. We don’t need to know the actual value of the improper integral, just whether it converges or diverges.

- To check if \( f \) is decreasing (at least eventually), it can help to check if \( f'(x) < 0 \) for large \( x \).

- If \( f \) is a function that is defined on \([1, \infty)\) but only continuous, positive, and decreasing on \([k, \infty)\), we can still apply the Integral Test. If \( \int_{k}^{\infty} f(x) \, dx \) converges, then the Integral Test says that \( \sum_{n=k}^{\infty} f(n) \) converges, so \( \sum_{n=1}^{\infty} f(n) \) also converges. Similarly, if \( f \) satisfies the conditions of the Integral Test on \([k, \infty)\) and \( \int_{k}^{\infty} f(x) \, dx \) diverges, then \( \sum_{n=1}^{\infty} f(n) \) also diverges. (This is because adding a finite number of terms to the series will not affect its convergence or divergence. See the previous section.)

- In general, the value of the integral \( \int_{k}^{\infty} f(x) \, dx \) is **not** the same as the value of the series \( \sum_{n=k}^{\infty} a_n \). The Integral Test doesn’t say anything about the value of the series \( \sum_{n=k}^{\infty} a_n \), just whether it converges or diverges.

- From the Integral Test, we get the p-test for series:
  - If \( p > 1 \), then \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) converges.
– If \( p \leq 1 \), then \( \sum_{n=1}^{\infty} \frac{1}{n^p} \) diverges.

- A series of the form \( \sum \frac{1}{n^p} \) is called a \( p \)-series.

- Suppose \( f \) is a function that satisfies the conditions of the Integral Test for \( x \geq n \) (\( f \) is continuous, positive, decreasing, \( f(k) = a_k \) for natural numbers \( k \)) and \( \sum a_k \) is convergent. Let \( s = \sum a_k \) and let \( R_n = s - s_n \), where \( s_n \) is the \( n \)th partial sum. Then
  \[
  \int_{n+1}^{\infty} f(x) \, dx < R_n < \int_{n}^{\infty} f(x) \, dx.
  \]

  We think of \( R_n \) as the error in estimating the value of the whole series \( s \) (which is \( \sum a_n \)) by the \( n \)th partial sum. The value of the integral \( \int_{n}^{\infty} f(x) \, dx \) gives us an upper bound for the error.

- If we add \( s_n \) to the inequalities in the previous equation we get:
  \[
  s_n + \int_{n+1}^{\infty} f(x) \, dx < s < s_n + \int_{n}^{\infty} f(x) \, dx.
  \]

11.4: The Comparison and Limit Comparison Tests

- **The Comparison Test:** Suppose \( \sum a_n \) and \( \sum b_n \) are series with positive terms, and suppose \( a_n \leq b_n \) for large \( n \). Then:
  - If \( \sum b_n \) converges, then \( \sum a_n \) converges.
  - If \( \sum a_n \) diverges, then \( \sum b_n \) diverges.

- If \( \sum b_n \) diverges, then this does not say anything about \( \sum a_n \); \( \sum a_n \) can converge or can diverge. If \( \sum a_n \) converges, then this does not say anything about \( \sum b_n \); \( \sum b_n \) can converge or can diverge.

- **The Limit Comparison Test:** Suppose \( \sum a_n \) and \( \sum b_n \) are series with positive terms. If
  \[
  \lim_{n \to \infty} \frac{a_n}{b_n} = c
  \]
  where \( 0 < c < \infty \) (i.e. \( c \) is a positive real number), then \( \sum a_n \) and \( \sum b_n \) both converge or both diverge.

- When using these tests, it is often helpful to compare to a \( p \)-series or geometric series.

- The following facts can also be helpful when trying to use the comparison test:
  - If \( a > 0 \), then for large \( n \), \( \ln(n) < n^a \). In other words, \( n \) raised to any positive power outgrows \( \ln(n) \) eventually.
  - If \( a > 0 \) and \( r > 0 \), then for large \( n \), \( e^{an} > n^r \). In other words, \( n \) raised to any positive power is outgrown by \( e^{an} \) eventually.
  - If \( a > 0 \), then for large \( n \), \( e^{an} < n! \) (where \( n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1 \)).

- Some useful comparisons with trig/inverse trig functions:
  - for all \( n \geq 1 \), \( 0 < \arctan(n) < \frac{\pi}{2} \)
  - for all \( n \geq 1 \), \( 0 \leq \sin^2(n) \leq 1 \) and \( 0 \leq \cos^2(n) \leq 1 \)

- Some useful facts about limits:
  - From Calculus I, we know that \( \lim_{x \to 0} \frac{\sin(x)}{x} = 1 \). Therefore, \( \lim_{n \to \infty} \frac{\sin(1/n)}{(1/n)} = 1 \). In fact, for any number \( p > 0 \), we have \( \lim_{n \to \infty} \frac{\sin(1/n^p)}{(1/n^p)} = 1 \).
  - If \( a > 0 \), then \( \lim_{n \to \infty} a^{1/n} = 1 \).
11.5: Alternating Series

Alternating Series Test:
Given an alternating series \( \sum (-1)^n b_n \), if

(i) \( 0 < b_{n+1} \leq b_n \) for all \( n \)
(ii) \( \lim_{n \to \infty} b_n = 0 \)

then the series \( \sum (-1)^n b_n \) converges.

Remainder estimate: If (i) and (ii) hold, then \( |R_n| \leq b_{n+1} \).

Ex: Give an example of an alternating series that converges and has value \( \frac{2}{3} \).

Ex: Show that the series \( \sum \frac{(-1)^{n-1}}{\sqrt{n+1}} \) converges. How many terms do you need to take to approximate the series within 0.01 of the true value?

Miscellaneous

- If you’re trying to determine if a series \( \sum a_n \) converges or diverges, it’s often helpful to first try the Test for Divergence. If \( \lim_{n \to \infty} a_n \neq 0 \), then you can say right away that the series diverges.

- The integral test is often harder to apply than the limit comparison test or comparison test, so use it if you can’t think of another way to evaluate convergence or divergence. Of course, on the exam, you should read problems carefully to check if you are being asked to use a specific test.

- Make sure you know what the graphs of the inverse trig functions \( \arctan(x) \), \( \arcsin(x) \), and \( \arccos(x) \) look like.

- \( \lim_{n \to \infty} \sin(n) \) does not exist. Neither does \( \lim_{n \to \infty} \cos(n) \).

Good luck!