Math 347 - Worksheet 2

Jan. 21, 2019

This worksheet is graded. Please divide yourselves in groups of 3-4 people. At the end of the class I expect that you turn in one whole proof written in a neat paper with the name of all members in your group. In case you get stuck in the question that I assigned you to solve ask me questions.

1) In this question you should use the following definition of the rational numbers.

**Definition.** A real number $x$ is *rational* if it may be written in the form $x = \frac{p}{q}$ where $p$ is an integer and $q$ is a positive integer; $x$ is *irrational* if it is not rational. Prove or disprove the following statements.

(i) If $x$ and $y$ are real numbers such that $3x + 5y$ is irrational, then at least one of $x$ and $y$ is irrational.

(ii) If $x$ and $y$ are rational numbers, then $3x + 4xy + 2y$ is rational.

(iii) If $x$ and $y$ are irrational numbers, then $3x + 4xy + 2y$ is irrational.

2) For $x$ and $y$ integer numbers, prove that $x^2 + y^2$ is even if and only if $x$ and $y$ have the same parity.

3) Prove that if $x$ and $y$ are positive real numbers, then $\sqrt{x + y} \neq \sqrt{x} + \sqrt{y}$. 
