1. Let \( f(x) = (1 - 3x^2)e^{3x^2} \).

(a) Find the Taylor series of \( f \) to fourth order (degree 4).

(b) Use Taylor series to find \( \lim_{x \to 0} \frac{\ln(f(x))}{x^4} \).

2. An object with mass \( m > 0 \) is dropped at time \( t = 0 \). Taking air resistance into account, its speed after \( t \) seconds is

\[
v = \frac{mg}{c}(1 - e^{-ct/m}),
\]

where \( c \) is a positive constant and \( g > 0 \) is the acceleration due to gravity.

a. Calculate \( \lim_{t \to 0} v \).

b. Calculate \( \lim_{t \to \infty} v \).

c. Calculate \( \frac{dv}{dt} \). Is it positive or negative?

d. Calculate \( \lim_{t \to \infty} \frac{dv}{dt} \).

e. Sketch \( v \) on the axes below, and indicate the behavior found in parts (a)—(d). In particular, on the \( v \)-axis show the values of \( v \) as \( t \to 0 \) and \( t \to \infty \).
3. An electric dipole has charges $\pm q$ separated by a distance $2a$. Here $q$ is constant. The electric field strength at position $r$ is $E = \frac{aq}{(a^2 + r^2)^{3/2}}$.

(a) Assume $r$ is constant. Find $\lim_{a \to \infty} a^2 E$.

(b) Suppose $a$ and $r$ depend on time $t$, with derivatives $r' = dr/dt$ and $a' = da/dt$. Find $E' = \frac{dE}{dt}$.

(Circle one answer. Show your work.)

A. $E' = \frac{-3aq(aa' + rr')}{(a^2 + r^2)^{5/2}}$

B. $E' = \frac{a'q(a^2 + r^2) - 3aq(aa' + rr')}{(a^2 + r^2)^{5/2}}$

C. $E' = \frac{(a'q + aq')(a^2 + r^2) - 3aq(aa' + rr')}{(a^2 + r^2)^{5/2}}$

D. $E' = \frac{a'q(4a^2 + r^2) + 3aqr r'}{(a^2 + r^2)^{5/2}}$

E. $E' = \frac{-3aqr r'}{(a^2 + r^2)^{5/2}}$

4. Gravel is dumped at a rate of 30 ft$^3$/min. It forms a conical pile with volume

$$V = \frac{1}{3} \pi r^2 h$$

where the radius $r$ of the base equals half the height $h$.

How fast is the height of the pile increasing, when the pile is 10 ft high?

5. A sample of ideal gas satisfies

$$PV = nRT$$

where $P$ is pressure, $V$ is volume, $n$ is the quantity of gas, $R$ is constant, and $T$ is temperature.

Use differentials to estimate the percentage change (that is, relative change) in volume that is required to keep the pressure constant, assuming the temperature increases by 3% and the quantity of gas decreases by 1%.