For each of the following, set up and solve an $\epsilon - \delta$ inequality which verifies the asserted limit.

1. \[ \lim_{x \to 2} 4x - 3 = 5 \]

   Start with 
   \[ |4x - 3 - 5| < \epsilon \]
   and simplify to 
   \[ |4x - 8| < \epsilon \]
   \[ 4|x - 2| < \epsilon \]
   \[ |x - 2| < \epsilon/4. \]

   So $\delta = \epsilon/4$ works.

2. \[ \lim_{x \to 0} x^2 = 0 \]

   Start with 
   \[ |x^2 - 0| < \epsilon \]
   and simplify 
   \[ |x|^2 < \epsilon \]
   \[ |x| < \sqrt{\epsilon} \]
   \[ |x - 0| < \sqrt{\epsilon} \]

   So $\delta = \sqrt{\epsilon}$ works.

3. \[ \lim_{x \to 4} x^2 = 16 \]

   We want 
   \[ |x^2 - 16| < \epsilon. \]

   Factorize: 
   \[ |(x + 4)(x - 4)| < \epsilon. \]

   This last inequality is what we want to show.

   We will use $|x - 4|$ to “control” $|x + 4|$. Notice that if we require $|x - 4| < 1$ then 
   \[ 3 < x < 5 \]
and adding 4 gives \[ 7 < x + 4 < 9 \]
and so \[ |x + 4| < 9, \]
so that \[ |(x + 4)(x - 4)| < 9|x - 4|. \]
We want the right-hand side to be less than \( \epsilon \). To achieve this we require (in addition to \( |x - 4| < 1 \) that \[ |x - 4| < \epsilon/9. \]
So \( \delta = \min(1, \epsilon/9) \) works.

4.

\[
\lim_{h \to 0} \frac{(1 + h)^2 - 1}{h} = 2
\]

Start with \[ \left| \frac{(1 + h)^2 - 1}{h} - 2 \right| < \epsilon \]
Use algebra to simplify:
\[
\left| \frac{1 + 2h + h^2 - 1}{h} - 2 \right| < \epsilon
\]
\[
\left| \frac{2h + h^2}{h} - 2 \right| < \epsilon
\]
\[
|2 + h - 2| < \epsilon
\]
\[
|h| < \epsilon
\]
\[
|h - 0| < \epsilon
\]
So \( \delta = \epsilon \) works.