Instructions for written homework.

- You are encouraged to work with others on these problems. You are expected to write the solutions yourself.
- Your solutions should be legible and well organized. Graders will deduct points for solutions which are not legible.
- Staple your pages together. Do not turn in notebook paper with tattered edges.
- Circle your answers (in those cases when it is appropriate to do so).

1. Let \( f(x) = 2x^2 + x + 1 \).
   (a) Find the Taylor polynomial of degree 2 about the point \( a = 1 \).
   (b) Multiply out \( T_2(x) \). Describe what you find, in one short sentence.

2. (a) Find the linear (first order) approximation to \( f(x) = \sin x \) about the point \( a = \pi/3 \).
   (b) Use your answer to approximate the value of \( \sin(1) \). How close is your answer to the actual value? (Use radians on your calculator!)
   (c) Repeat using the third order approximation to \( f(x) = \sin x \) about the point \( a = 0 \). Is this better or worse than the approximation from part (b)?

3. Approximate the value of \( e = e^1 \) using the first few terms of the Taylor series about 0. How many terms do you need, in order to obtain 3 correct decimal places of \( e \)?

4. Find the Taylor series for \( f(x) = \sin x \) about the point \( a = \pi/2 \). Your answer should give at least the first three non-zero terms of the series (you may leave the end of the series in the form \( \cdots + H.O.T \)).

5. One engineer says to another:
   "whenever I see \( \sqrt{1 + x} \) in a formula, I replace it with \( 1 + \frac{1}{2}x \)."
   (a) Explain why the engineer does this, in one short sentence.
   (b) Using a calculator or a graph, find a number \( \delta > 0 \) such that if \(|x| < \delta \) then the difference between \( \sqrt{1 + x} \) and \( 1 + \frac{1}{2}x \) is less than 0.1 in magnitude.

6. Use algebraic manipulation and known Taylor series from class to write down Taylor series about 0, for the following functions.
   **You do not need to take any derivatives to do these problems.**
   - Your answer should give at least the first three non-zero terms of each series. For example, we would write \( e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \text{H.O.T.} \). You do not need to use "\( \sum \)" notation.
     (a) \( f(x) = \cosh(x) \) (this "hyperbolic cosine" function is defined on page 254)
     (b) \( f(x) = \sin(4x) \)
     (c) \( f(x) = e^{-x^2} \)
     (d) \( f(x) = \frac{\sin x}{x} \)