Integration by Partial Fractions

Goal: evaluate integrals of the form

\[ \int \frac{P(x)}{Q(x)} \, dx \]

where \( P \) and \( Q \) are polynomials.
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**Step 0:** Reduce to the case \( \text{deg}(P) < \text{deg}(Q) \) by long division.

**Step I:** Factor \( Q \) into linear and irreducible quadratic terms.

**Step II:** Express the proper rational function \( \frac{R(x)}{Q(x)} \) as a sum of partial fractions.

**Case (i):** \( Q \) is a product of distinct linear factors.

**Case (ii):** \( Q \) contains irreducible quadratic factors, none of which are repeated.

**Case (iii):** One or more of the factors in \( Q \) is repeated.

**Step III:** Integrate each term in the partial fraction representation separately.
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